离散数学（011122）

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- **2.1 Basic Concepts of Propositional Logic**
- 2.2 Propositional logic equivalence transformations
- 2.3 Logical normal form

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2.1.1 Propositions and Connectives

- Propositions and Truth Values (Simple Propositions, Compound Propositions)
- Connectives (¬, ∧, ∨, →, ↔)

2.1.2 Propositional Formulas and Their Classification

- Propositional Formulas and Their Assignments
- Truth Tables
- Classification of Propositional Formulas

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- **Proposition**: A statement that can be judged as true or false.
- **Truth value of a proposition**: The result of the judgment, either true or false.
- **True proposition**: A proposition with a truth value of true.
- **False proposition**: A proposition with a truth value of false.

Note: Exclamatory sentences, imperative sentences, and interrogative sentences are not propositions. Also, paradoxes in declarative sentences or those with indeterminate judgment results are not propositions.

e.g. Example: Which of the following sentences are propositions?

(1) The capital of the People's Republic of China is Beijing. True proposition

(2) 2 + 3 =6. False proposition

(3) x + y > 8. Undecided True Value

(4) Can you play tennis? Interrogative sentences

(5) There is life on planets other than Earth.. Indeterminate judgment results

(6) This ! Exclamatory sentences

(7) Please close the door! Imperative sentences

(8) I am lying. Paradoxes

(1), (2), (5) are propositions. (3), (4), (6)~(8) are not propositions.

- **Simple Proposition (Atomic Proposition):** A proposition formed by simple statements.
  - **Symbolization of Simple Propositions:** Symbolization of simple propositions: Represented by $p, q, r, \ldots, p_i, q_i, r_i$ (where $i \geq 1$). '**1**' represents true, and '**0**' represents false.
- **Compound Proposition:**
  A statement formed by connecting simple propositions with logical connectives.
  Examples:

(1) If the weather is good tomorrow, we will play ball.
Let ***p***: The weather will be good tomorrow, and ***q***: We will play ball. **If *p*, then *q*.**

(2) Luffy is drinking milk tea and scrolling through his phone.
Let ***p***: Luffy is drinking milk tea, and ***q***: Luffy is scrolling through his phone. ***p* and *q*.**

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- **Definition 2.1:**"Non-*p*" (or "the negation of *p*") is called the ***negation*** of *p*, denoted as **¬*p***. The symbol ¬ is the negation connective, and it is defined such that **¬*p*** is true if and only if p is false.

  Example:
  - ***p***: 2 is a composite number. **¬*p***: 2 is not a composite number.
  - Since 2 is actually a prime number, ***p*** is false, and therefore **¬*p*** is true.

- **Definition 2.2:**

  "***p*** and ***q***" (or "***p*** with ***q***") is called the ***conjunction*** of p and q, denoted as ***p***∧***q***. The symbol ∧ is the conjunction connective, and it is defined such that ***p***∧***q*** is true if and only if both ***p*** and ***q*** are true simultaneously.

  Example:
  - ***p***: 2 is an even number. ***q***: 2 is a prime number.
  - ***p***∧***q***: 2 is an even prime number.
  - Since 2 is indeed both even and prime,

  both ***p*** and ***q*** are true, so ***p***∧***q*** is also true

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Symbolize the following propositions.

(1) Wang Xiao is smart and hardworking.

(2) Wang Xiao is not only smart but also hardworking.

(3) Wang Xiao is smart, but not hardworking.

(4) Wang Xiao is not unintelligent, but rather not hardworking.

(5) Both Zhang Hui and Wang Li are outstanding students.

(6) Zhang Hui and Wang Li are classmates

(1) $p \wedge q$

(Let ***p***: Wang Xiao is smart, ***q***: Wang Xiao is hardworking.)

(2) $p \wedge q$

(3) $p \wedge \neg q$

(4) $\neg(\neg p) \wedge \neg q$

(5) $r \wedge s$

(Let *r*: Zhang Hui is an outstanding student, *s*: Wang Li is an outstanding student.)

(6) t (A simple proposition, "and" connects two nouns, the entire sentence is a simple proposition.)

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- **Definition 2.3: "*p* or *q*"** is called the disjunctive form of ***p*** and ***q***, denoted as ***p***∨***q***. The symbol ∨ is called ***the Disjunction connective***, and ***p***∨***q*** is false if and only if both ***p*** and ***q*** are false.

e.g.»»Example: Wang Yan has studied English or French.

Let ***p***: Wang Yan has studied English,

q: Wang Yan has studied French.

Symbolized as ***p***∨***q***.

- The English term for "∨" is "inclusive" (Inclusive), corresponding to the "or" in everyday language. The English term for "∧" is "exclusive" (Exclusive), corresponding to the "and" in everyday language. "**Inclusive or**" and "**exclusive or**" are their combined forms.

e.g. Example: This task is to be done by either Zhang San or Li Si. Let ***p***: Zhang San does this task, ***q***: Li Si does this task. It should be symbolized as:

(1) $(p \wedge \neg q) \vee (\neg p \wedge q)$.

(2) $p \oplus q$

(3) P XOR q

e.g.»» Example: Symbolize the following propositions:

(1) 2 is a prime number or 4 is a prime number.

(2) 2 is a prime number or 3 is a prime number.

(3) 4 is a prime number or 6 is a prime number.

Solve: Let:

p: 2 is a prime number,
q: 3 is a prime number,
r: 4 is a prime number,
s: 6 is a prime number.

(1) $p \vee r$, 1 ∨ 0 = 1

(2) $p \vee q$, 1 ∨ 1 = 1

(3) $r \vee s$, 0 ∨ 0 = 0

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e.g. Example: Symbolize the following propositions:

(4) Yuan Yuan can take an apple or a pear.

Solve: Let:

t: Yuan Yuan takes an apple,

u: Yuan Yuan takes a pear.

$$(t \wedge \neg u) \vee (\neg t \wedge u)$$

(5) Wang Xiaohong was born in 1975 or 1976

Solve: Let:

v: Wang Xiaohong was born in 1975,

w: Wang Xiaohong was born in 1976.

$$(v \wedge \neg w) \vee (\neg v \wedge w)$$

Can logical expression "$(v \wedge \neg w) \vee (\neg v \wedge w)$" be converted to "$v \vee w$"?